Hurewicz spaces in the Laver model

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In L there exists a co-analytic Menger subspace of ω^ω which is not σ-compact.

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More generally: $\mathfrak{b}$-Sierpinski sets are Hurewicz and $\mathfrak{d}$-Luzin sets are Menger.

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Fact. There exists a $\mathfrak{d}$-concentrate set.

Proof. Fix a dominating $\{d_\alpha : \alpha < \mathfrak{d}\} \subset \omega^\omega$ and inductively construct $S = \{s_\alpha : \alpha < \mathfrak{d}\} \subset \omega^{\uparrow\omega}$ such that $s_\alpha \not\leq^* d_\beta$ for all $\beta \leq \alpha$.

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Is there a ZFC example of a space M such that $\mathcal{K}(M)$ is hereditarily Baire non-Polish?

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Let P, Q be directed posets. Then $P \geq_T Q$ if there is a map $\phi : P \to Q$ that takes cofinal subsets of P to cofinal subsets of Q.

Theorem (Christensen 1974)

If M is a separable metrizable space, then $\omega^\omega \geq_T \mathcal{K}(M)$ if and only if M is Polish.

Theorem (Gartside-Mamatelashvili 201?)

Let M be a separable metrizable space. Then $\mathcal{K}(M) \not\geq_T \mathcal{K}(\mathbb{Q})$ iff $\mathcal{K}(M)$ is hereditarily Baire.

Question

Is there a ZFC example of a space M such that $\mathcal{K}(M)$ is hereditarily Baire non-Polish?

Yes, by the following

Theorem (Gartside-Medini-Z. 2016)

Let $X \subset 2^\omega$ be Menger non-σ-compact. Then $\mathcal{K}(2^\omega \setminus X)$ is hereditarily Baire non-Polish.

Question (Fremlin 1991)

Is it consistent that there exists an analytic non-Borel $X \subset 2^\omega$ *such that* $\mathcal{K}(\mathbb{Q}) \geq_T \mathcal{K}(X)$*?*

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We construct a co-analytic Hurewicz $Y \subset 2^\omega$ such that $X = 2^\omega \setminus Y$ is as required. We use results of Vidnyanszky to make sure that Y is co-analytic, which extend and unify earlier results of A. Miller.

Preservation by products

Fact. (CH.) There are two Sierpinski (hence Hurewicz) sets S_0, S_1 whose product is not Menger.

Proof. Fix a countable dense $Q \subset 2^\omega$ and write $2^\omega \setminus Q = \{x_\alpha : \alpha < \omega_1\}$.

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Problem

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Note: The conclusion doesn't follow from the Borel's Conjecture.

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Lemma

- *In the Laver model every Hurewicz subspace of $\mathcal{P}(\omega)$ is weakly concentrated.*
- *If $\mathfrak{b} > \omega_1$, then a product of a weakly concentrated $X \subset 2^\omega$ and a Hurewicz $Y \subset 2^\omega$ is Menger.*

How concentration works in products

Time permitting, it should be explained on the blackboard why Hurewicz x concentrated is Menger.

The last slide

Thank you for your attention.