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Conjectures of Rado and Chang and the Strong Tree Property

Joint work with Liuzhen Wu

Jech introduced a strengthening of the Tree Property, now called the Strong Tree Property. He noticed that, analogous to the relationship between the notions of the Tree Property and a weakly compact cardinal, an inaccessible cardinal κ has the Strong Tree Property if and only if κ is strongly compact. Baumgartner showed that PFA implies $\aleph_2$ has the Tree Property, and recently Weiß proved that PFA indeed implies $\aleph_2$ has the Strong Tree Property.

Rado's Conjecture, RC, is the following principle: A family of convex sets of a linearly ordered set is the union of countably many disjoint subfamilies if and only if each of its subfamilies of size $\aleph_1$ is also the union of countably many disjoint subfamilies. Todorcevic proved the consistency of RC via a large cardinal. Similarly to well-known forcing axioms like MM or PFA, RC has interesting implications such as the SCH, $\mathfrak{c} \leq \aleph_2$, Chang's Conjecture, etc. However, RC implies $\neg\mathrm{MA}_{\omega_1}$.

RC is consistent either with CH or $\neg\mathrm{CH}$, but CH implies the negation of the Tree Property for ω_2. Therefore, to obtain a result between RC and the Strong Tree Property for ω_2 similar with the above mentioned forcing axioms, we need at least to assume the negation of the Continuum Hypothesis. We show that this condition is not only necessary but sufficient. We do it via a strong version of Chang's Conjecture, CC^*, which is a consequence of RC and MM. We have the following:

Theorem. $\mathrm{CC}^* + \neg\mathrm{CH}$ *imply* ω_2 *has the Strong Tree Property.*

[1] Torres-Pérez, V., Wu, L., Strong Chang's Conjecture, Semi-Stationary Reflection, the Strong Tree Property and two cardinals Square Principles. To appear in Fundamenta Mathematicae.