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Additivity of the ideal of microscopic sets

Following J. Appell, we call a set $M \subseteq \mathbb{R}$ *microscopic* if for each $\varepsilon > 0$ there exists a sequence of intervals $(J_n)_{n\in\omega}$ covering M and such that $|J_n| \leq \varepsilon^{n+1}$ for each $n \in \omega$ (cf. [1]).

During the talk I will consider the family $\mathcal{M}$ of all microscopic sets as well as two of its generalizations – families of picoscopic sets ($\mathcal{P}$) and nanoscopic sets ($\mathcal{N}$). In particular, I will show that, in contrast to $\mathcal{M}$, families $\mathcal{N}$ and $\mathcal{P}$ do not form ideals. This will lead us to the main result showing (in ZFC) that the additivity of $\mathcal{M}$ is exactly ω_1.

[1] J. Appell, *Insiemi ed operatori "piccoli" in analisi funzionale*, Rend. Instit. Mat. Univ. Trieste, **33**, (2001), 127–199.