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Mathias forcing and generic ultrafilters

I will analyze the Mathias–Prikry forcing $M(\mathcal{F})$ where $\mathcal{F}$ is a filter on ω and derive a condition for reals which is sufficient for genericity in this poset. As an application, I will present a characterization of certain ultrafilters on ω generic over $L(\mathbb{R})$ (due to J. Zapletal).

Theorem. *Suppose there are large cardinals. Let $\mathcal{I}$ be an F_σ on ω and $\mathcal{U}$ be an ultrafilter on ω. The following are equivalent:*

1. *$\mathcal{U}$ is Q-generic over $L(\mathbb{R})$ where Q is the poset of $\mathcal{I}$-positive sets ordered by modulo finite inclusion,*

2. *$\mathcal{U}$ is a P-point, $\mathcal{U} \cap \mathcal{I} = \emptyset$, and whenever $E \subset \mathcal{P}(\omega)$ is a closed set disjoint from $\mathcal{U}$, then there is a set $e \in \mathcal{P}(\omega) \setminus \mathcal{U}$ such that $E \subset \langle \mathcal{I}, \{ e \} \rangle$.*