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Book of Abstracts

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Foreword

Novi Sad Conference in Set theory and General Topology (SetTop) is organized biannually on the Department of Mathematics of The Faculty of Science at the University of Novi Sad. This is its second edition, succeeding LogTop 2012, organized as one of three conferences celebrating 50 years of The Seminar for analysis and foundation of mathematics, led by Professor Bogoljub Stanković.

This event came as a natural consequence of the growing number of people on the Department of Mathematics interested in set theory and general topology, and their wish to become more connected to scientists around the world working in the same areas.

The main topics of this year's conference are set theory, model theory and general topology.

We hope that you have a pleasant stay in Novi Sad!

The organizing committee

Invited lectures

Dana Bartošová	Sao Paulo
Piotr Borodulin-Nadzieja	Wroclaw
Jan van Mill	Amsterdam
Justin Moore	Ithaca, NY
Stevo Todorčević	Paris - Toronto

Participants

Giorgio Audrito	Torino	**Boriša Kuzeljević**	Beograd
Bojan Bašić	Novi Sad	**Rozalia Madarasz**	Novi Sad
Lev Bukovsky	Košice	**Dragan Mašulović**	Novi Sad
Nela Cicmil	Oxford	**Nenad Morača**	Novi Sad
Jana Flašková	Pilsen	**Aleksandar Pavlović**	Novi Sad
Milan Grulović	Novi Sad	**Dušan Radičanin**	Novi Sad
Gabriele Gullà	Roma	**Robert Rałowski**	Wroclaw
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Dejan Ilić	Beograd	**Boris Šobot**	Novi Sad
István Juhász	Budapest	**Thilo Weinert**	Jerusalem
Miloš Kurilić	Novi Sad	**Szymon Zeberski**	Wroclaw

ABSTRACTS

Giorgio Audrito

University of Torino, Italy
giorgio.audrito@gmail.com

Absoluteness via Resurrection

Joint work with Matteo Viale

Generic absoluteness over a theory T containing ZFC is the phenomena by which the truth value of mathematical statements of a certain logical complexity is invariant with respect to appropriate types of forcing which preserve T. This topic has been studied since the introduction of forcing in the late '60, and is motivated by the broad success that the method of forcing reported on consistency results.

These kind of results for a theory T provide a mean to restrict the independence phenomena dating back to Gödel's incompleteness theorems, and can be used to turn the consistency proofs of certain first order statements ϕ into actual derivations (in first order calculus) of ϕ from T.

We expand on Viale and Hamkins work (among others) on generic absoluteness and resurrection axioms and we introduce the iterated resurrection axioms $\mathrm{RA}_\alpha(\Gamma)$ as α ranges among the ordinals and Γ varies among various classes of forcing notions. Our main results (obtained jointly with Viale) are the following:

Theorem. *If $\mathrm{RA}_\omega(\Gamma)$ holds and $\mathbb{B} \in \Gamma$ forces $\mathrm{RA}_\omega(\Gamma)$, then $H_\mathfrak{c}^V \prec H_\mathfrak{c}^{V^\mathbb{B}}$ (where $\mathfrak{c} = 2^{\aleph_0}$ is the continuum as computed in the corresponding models).*

Hence a statement $\phi^{H_\mathfrak{c}}$ regarding the structure $H_\mathfrak{c}$ is first order derivable in the theory $T = \mathrm{ZFC} + \mathrm{RA}_\omega(\Gamma)$ whenever T proves its consistency together with T by means of a forcing in Γ.

Theorem. $\mathrm{RA}_\alpha(\Gamma)$ *is consistent relative to the existence of a Mahlo cardinal for the following classes of posets: all, ccc, axiom-A, proper, semiproper.*

We remark that the existence of a Mahlo cardinal is very low in the large cardinal hierarchy.

Theorem. $\mathrm{RA}_\alpha(\Gamma)$ *for* Γ *the class of stationary set preserving posets is consistent relative to the existence of a stationary limit of supercompact cardinals.*

In the talk we shall motivate the foundational role played by generic absoluteness results, sketch a proof of some of our results, and compare our results to the current literature in the field.

Dana Bartošová

University of Sao Paulo, Brasil
dana.bartosova@gmail.com

Lelek fan and generalizations of finite Gowers' FIN$_k$ theorem

Joint work with Aleksandra Kwiatkowska

We describe a one-dimensional continuum, known as the Lelek fan, as a natural quotient of a projective Fraïssé limit of a class of finite ordered rooted trees. This allows us to use model-theoretic means to study the Lelek fan as well as its group of homeomorphisms. Striving for a computation of the universal minimal flow of the group of homeomorphisms of the Lelek fan, we find an extremely amenable subgroup of the homeomorphism group, which requires us to generalize the finite FIN$_k$ theorem of Gowers.

Bojan Bašić

Department of Mathematics and Informatics, Faculty of Sciences,
University of Novi Sad, Serbia
bojan.basic@dmi.uns.ac.rs

The Algebra $\mathbb{B}(\mathcal{O})$

Joint work with Miloš S. Kurilić

Given a topological space $\langle X, \mathcal{O} \rangle$, we define a Boolean algebra $\mathbb{B}(\mathcal{O})$ as a Boolean analogue of the well-known Borel σ-algebra. We study different properties of the algebra $\mathbb{B}(\mathcal{O})$. Our main result presents a necessary and sufficient condition for a given Boolean algebra $\mathbb{B}$ to be isomorphic to the algebra $\mathbb{B}(\mathcal{O})$ for some topological space $\langle X, \mathcal{O} \rangle$.

Piotr Borodulin-Nadzieja

Mathematical Institute, University of Wroclaw, Poland
Piotr.Borodulin-Nadzieja@math.uni.wroc.pl

Measures on Suslinean spaces

A compact Hausdorff space is Suslinean if it is ccc and non-separable. We will overview some classical examples of small Suslinean spaces and discuss the problem when a Suslinean space can serve as a support for a measure.

Lev Bukovský

Institute of Mathematics, Faculty of Sciences,
P. J. Šafárik University, Košice, Slovakia
lev.bukovsky@upjs.sk

Generic Extensions of Models of ZFC

We present a new proof of the result of [B2]

Theorem *Let* $M_1 \subseteq M_2$ *be models of* **ZFC** *with same ordinals,* κ *being an uncountable regular cardinal in* M_1*. Then* M_2 *is a* κ*-C.C. generic extension of* M_1 *if and only if for every function* $f \in M_2$*,* $dom(f) \in M_1$*,* $rng(f) \subseteq M_1$ *there exists a function* $g \in M_1$ *such that* $f(x) \in g(x)$ *and* $|g(x)|^{M_1} < \kappa$ *for every* $x \in dom(f)$*.*

The proof is based on the following fundamental result

Lemma *If* $Apr_{M_1,M_2}(\kappa)$ *holds true then for any set* $a \in M_2$*,* $a \subseteq M_1$*, the model* $M_1[a]$ *is a generic extension of* M_1*.*

The proofs of the lemma in [B2] and in [FFS] essentially differ and are different from the presented one. Presented proof is based on an immediate strengthening of the result of [B1], which is actually a special case of the lemma.

[B1] Bukovský L., *Ensembles génériques d'entiers,* C.R. Acad. Sc. Paris, **273** (1971), 753–755.

[B2] Bukovský L., *Characterization of generic extensions of models of set theory,* Fund. Math., **83** (1973), 35–46.

[FFS] Friedman S.D., Fuchino S. and Sakai H., *On the set-generic multiverse*, preprint.

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Jana Flašková

University of West Bohemia, Pilsen, Czech Republic
flaskova@kma.zcu.cz

Van der Waerden spaces and their relatives

A set of natural numbers which contains arithmetic progressions of arbitrary length is called an AP-set. According to the van der Waerden theorem sets which are not AP-sets form an ideal which is usually denoted as van der Waerden ideal. A topological space X is called van der Waerden space if for every sequence $\langle x_n \rangle_{n \in \mathbb{N}}$ in X there exists a converging subsequence $\langle x_{n_k} \rangle_{k \in \mathbb{N}}$ so that $\{n_k : k \in \mathbb{N}\}$ is an AP-set, i.e. the set is positive with respect to the van der Waerden ideal.

We investigate the classes of topological spaces which are defined by replacing the van der Waerden ideal in the definition of van der Waerden spaces by another suitable ideal on the natural numbers such as the summable ideal $\mathcal{I}_{1/n}$. We are interested in inclusions between such classes of spaces and we consider their topological properties (e.g. productivity). Some examples of such spaces with some additional properties are obtained as Ψ-spaces for some particular almost disjoint families.

Dejan Ilić

Faculty of Transport and Traffic Engineering,
University of Belgrade, Serbia
d.ilic@sf.bg.ac.rs

On small expansions of $(\omega, <)$ and $(\omega+\omega^*, <)$

An *expansion* of a first-order structure is any structure obtained from it by adding additional finitary relations and functions. Two first-order structures are *definitionally equivalent* iff they have the same domain and the same definable sets. An expansion is *definitional* if it is definitionally equivalent to original structure. An expansion is *essentially unary* if it is definitionally equivalent to expansion which is obtained by adding only unary relations.

We investigate expansions of either $(\omega, <)$ or $(\omega+\omega^*, <)$, requiring that the complete first-order theory is small: there are only countably many complete types without parameters. The main result is next theorem.

Theorem: Let $\mathcal{M}$ be expansion of either $(\omega, <)$ or $(\omega + \omega^*, <)$ such that Th($\mathcal{M}$) is small and CB$(x = x) = 1$. Then $\mathcal{M}$ is an essentially unary expansion. □

Fist-order theory T is binary if for every formula $\varphi(x_0, \ldots, x_n)$ there is a formula $\psi(x_0, \ldots, x_n)$, a Boolean combination of formulas in at most two variables such that φ and ψ are equivalent modulo T.

Galvin proved that every theory of linear order is binary (Theorem 13.37 in Rosenstein's *Linear Orderings*, Academic Press New York 1982). Linear ordering with at most countable many unary predicates can be 'coded' in pure linear ordering, so its theory is also binary. It follows that if $\mathcal{M}$ is an expansion of $(\omega, <)$ or $(\omega+\omega^*, <)$ such that CB$(x = x) = 1$, then Th($\mathcal{M}$) is binary. This conclusion led to following conjecture.

Conjecture: If $\mathrm{Th}(\omega, <, \ldots)$ is small, then it is binary.

We will discuss this conjecture and give some partial results.

István Juhász

Alfréd Rényi Institute, Budapest, Hungary
juhasz.istvan@renyi.mta.hu

Discrete subspaces of countably compact spaces

In the first part of this talk I present the following result that is joint with S. Shelah: THEOREM. For every infinite cardinal κ there is a κ-bounded 0- dimensional T_2 space with a *discretely untouchable* point, i.e. a nonisolated point to which no discrete set accumulates. In the second part, whose results are joint with L. Soukup and Z. Szentmilóssy, I deal with ωD*-bounded spaces*, that is T_1-spaces in which the closure of any *countable discrete* set is compact. Here are some of our main results:

- Regular ωD-bounded spaces of Lindelöf degree ¡ $cov(M)$ are ω-bounded.
- If $\mathfrak{b} > \omega_1$ then regular ωD-bounded spaces of countable tightness are ω-bounded. But under CH there is a first countable and locally compact T_2, hence regular, space that is not ω-bounded.
- If a product of Hausdorff spaces is ωD-bounded then all but one of its factors must be ω-bounded.
- Any product of at most $\mathfrak{t}$ many Hausdorff ωD-bounded spaces is countably compact.

Miloš S. Kurilić

Department of Mathematics and Informatics, Faculty of Sciences,
University of Novi Sad, Serbia
milos@dmi.uns.ac.rs

Posets of copies, embedding monoids, and interpretability of relational structures

We consider several "similarity relations" between relational structures:

- equality, isomorphism, equimorphism, several forms of bi-interpretability,
- some similarities of their embedding monoids $\mathbb{E}\mathrm{mb}(\mathbb{X})$ and the corresponding Green's preorders, which can be isomorphic, can have Boolean completions isomorphic, etc.,
- some similarities of their posets of isomorphic substructures $\langle \mathbb{P}(\mathbb{X}), \subset \rangle$, which can be equal, isomorphic, forcing equivalent etc.

Clearly, all such similarities are equivalence relations on the class of relational structures and some results concerning the hierarchy of these equivalences and the corresponding classifications of structures will be presented. For example, if $\mathbb{X}$ and $\mathbb{Y}$ are structures (of possibly different languages and size), then

$\mathbb{X} \cong \mathbb{Y}$

$\Rightarrow$ $\mathbb{X}$ and $\mathbb{Y}$ are quantifier-free bi-interpretable without parameters

$\Rightarrow$ $\mathbb{E}\mathrm{mb}(\mathbb{X}) \cong \mathbb{E}\mathrm{mb}(\mathbb{Y})$

$\Rightarrow$ $\langle \mathrm{Emb}(\mathbb{X}), \preceq^R_X \rangle \cong \langle \mathrm{Emb}(\mathbb{Y}), \preceq^R_Y \rangle$ (right Green's pre-orders)

$\Rightarrow$ $\langle \mathbb{P}(\mathbb{X}), \subset \rangle \cong \langle \mathbb{P}(\mathbb{Y}), \subset \rangle$

$\Rightarrow$ $\mathrm{ro}\,\mathrm{sq}\langle \mathbb{P}(\mathbb{X}), \subset \rangle \cong \mathrm{ro}\,\mathrm{sq}\langle \mathbb{P}(\mathbb{Y}), \subset \rangle$ (Boolean completions).

More generally, if two structures $\mathbb{X}$ and $\mathbb{Y}$ are quantifier-free bi-interpretable, then the corresponding reversed Green's pre-orders are forcing equivalent. As an application of these results we show that if $\mathbb{X}$ is a countable reflexive or irreflexive ultrahomogeneous binary structure which is not biconnected, then $\mathbb{P}(\mathbb{X}) \cong \mathbb{P}(\mathbb{Z})^n$, for some biconnected ultrahomogeneous digraph $\mathbb{Z}$ and some $n \geq 2$, or $\operatorname{sq} \mathbb{P}(\mathbb{X})$ is an atomless and ω_1-closed poset; thus, under CH, $\operatorname{ro} \operatorname{sq} \mathbb{P}(\mathbb{X}) \cong P(\omega)/\operatorname{Fin}$.

Boriša Kuzeljević

Mathematical Institute SANU, Belgrade, Serbia
borisa@turing.mi.sanu.ac.rs

Maximal chains of isomorphic substructures of ultrahomogeneous relational structures

Joint work with Miloš Kurilić

We present some general results on maximal chains of isomorphic substructures of countable relational structures, and apply these results to completely describe the maximal chains of isomorphic substructures of countable ultrahomogeneous graphs and countable ultrahomogeneous partial orders.

Dragan Mašulović

Department of Mathematics and Informatics, Faculty of Sciences,
University of Novi Sad, Serbia
dragan.masulovic@dmi.uns.ac.rs

A remark on a general nature of the Katětov construction

Joint work with Wiesław Kubiś

In his 1988 paper "On universal metric spaces" M. Katětov published a new construction of the Urysohn space as the limit of the chain of embeddings $X \hookrightarrow K(X) \hookrightarrow K^2(X) \hookrightarrow K^3(X) \hookrightarrow \cdots$. Building on the observation that the construction $K(X)$ is functorial, in this talk we show that there is more behind this construction than meets the eye. We actually show that the same strategy applies to the construction of a wide class of Fraïssé limits, one of which is the rational Urysohn space, of course.

Jan van Mill

University of Amsterdam, Netherlands
j.van.mill@vu.nl

Every ccc-pseudocompact crowded space is resolvable

A space X is called *resolvable* if it has two disjoint dense subsets. Observe that a resolvable space is *crowded*, i.e., has no isolated points. A space is called *irresolvable* if it is not resolvable. The notion of resolvability is due to Hewitt and Ceder, respectively. It is known that every locally compact crowded space is resolvable. It is also known that there are irresolvable crowded spaces (Hewitt). Kunen, Szymański and Tall proved assuming $V = L$, that every crowded Baire space is resolvable. Moreover, they showed that if ZFC is consistent with the existence of a measurable cardinal, then ZFC is consistent with the existence of an irresolvable (zero-dimensional) Baire space. It was shown in Comfort and García-Ferriera that every countably compact crowded space is resolvable. It was asked by them whether every pseudocompact crowded space is resolvable. Since every pseudocompact space is Baire, the answer is yes if one assumes $V = L$. We prove here that every pseudocompact crowded space which satisfies the countable chain condition (abbreviated: ccc) is resolvable.

Justin Moore

Cornell University, Ithaca, NY, USA
justin@math.cornell.edu

Amenability and groups of homeomorphisms

Is Thompson's group F amenable? This talk will not answer this question but aims to shed some light on how methods and intuition from ultrafilter dynamics and Ramsey theory may be useful in its solution. I will also discuss recent joint work with Yash Lodha in which we construct a group sharing many of F's properties which is non amenable.

Nenad Morača

Department of Mathematics and Informatics, Faculty of Sciences,
University of Novi Sad, Serbia
nenad.moraca@dmi.uns.ac.rs

The condensation order on $\mathrm{Rel}(X)$

Joint work with Miloš Kurilić

The topic of this paper are relational structures of the form $\langle X, \rho\rangle$, where $\rho \in \mathrm{Rel}(X) := P(X \times X)$. We define an equivalence relation $\sim_c$ on $\mathrm{Rel}(X)$ called the condensation equivalence, such that $[\rho]_{\sim_c}$ is the convex envelope of $[\rho]_{\cong}$, and a partial order $\leq$ on the quotient $\mathrm{Rel}(X)/\sim_c$ called the condensation order. We classify relations based on the characteristics of their equivalence classes in the poset $\langle \mathrm{Rel}(X), \subset\rangle$, and solve the problem of the cardinality of the classes in the case when X is countable. Next, we introduce suborders $D_\rho = \{[\rho \cup \Delta_A]_{\sim_c} : A \subset X\}$ for irreflexive ρ, and study their properties in the correlation with the properties of the automorphism group $\mathrm{Aut}\langle X, \rho\rangle$.

Aleksandar Pavlović

Department of Mathematics and Informatics, Faculty of Sciences,
University of Novi Sad, Serbia
apavlovic@dmi.uns.ac.rs

On the sequence convergence of the Cantor and Aleksandrov cube on an arbitrary complete Boolean algebra

Joint work with Miloš Kurilić

For a sequence $x = \langle x_n : n \in \omega \rangle$ and the point $a \in 2^\kappa$ let $X_n = x_n^{-1}[\{1\}]$, and $A = a^{-1}[\{1\}]$. A sequence x converges to the point a in the Cantor cube iff $\bigcup_{k\in\omega}\bigcap_{n\geq k} X_n = \bigcap_{k\in\omega}\bigcup_{n\geq k} X_n = A$, and in the Aleksandrov cube iff $\bigcap_{k\in\omega}\bigcup_{n\geq k} X_n \subseteq A$.

Defining sequence convergence on an arbitrary complete Boolean algebra by replacing intersection and union with meet and join, and taking the maximal topology preserving this convergence, we obtain the generalizations of the Cantor and Aleksandrov cube on an arbitrary complete Boolean algebra.

It is known that the union of the topology on Aleksandrov cube and its algebraic dual is a subbase for the Cantor cube, and a sequence converges in the Cantor cube to a point a iff it converges to a in the Alexandrov cube and its dual.

We prove that both of these properties hold in the class of Maharam algebras (ccc and weakly distributive), the second one holds in the class of algebras satisfying condition $(\hbar)$ (which follows from $\mathfrak{t}$-cc and implies $\mathfrak{s}$-cc), and, using a notion of base matrix tree, we define a Boolean algebra and a sequence in it which witness that those properties do not hold in general.

Robert Ratowski

Wrocław University of Technology, Poland
robert.ralowski@pwr.edu.pl

Group action on Polish spaces

In this paper we investigate the action of Polish groups (not necessary abelian) on an uncountable Polish spaces. We consider two main situations. First, when the orbits given by group action are small and the second when the family of orbits are at most countable. We have found some subgroups which are not measurable with respect to a given σ-ideals on the group and the action on some subsets gives a completely nonmeasurable sets with respect to some σ-ideals with a Borel base on the Polish space. In most cases the general results are consistent with ZFC theory and are strictly connected with cardinal coefficients. We give some suitable examples, namely the subgroup of isometries of the Cantor space where the orbits are suffitiently small. In a opposite case we give an example of the group of the homeomorphisms of a Polish space in which there is a large orbit and we have found the subgroup without Baire property and a subset of the mentioned space such that the action of this subgroup on this set is completely nonmeasurable set with respect to the σ-ideal of the subsets of first category.

Boris Šobot

Department of Mathematics and Informatics, Faculty of Sciences,
University of Novi Sad, Serbia
sobot@dmi.uns.ac.rs

Number theory in the Stone-Čech compactification

In the book "Algebra in the Stone-Čech compactification" by Hindman and Strauss it is described how a semigroup operation on a discrete topological space S can be extended to its Stone-Čech compactification βS, and many properties of the obtained semigroup are established.

There are several possible ways to extend the divisibility relation on the set N of natural numbers to βN. The most appropriate one is defined as follows: for $p, q \in \beta N$, $p \mid_R q$ if there is $r \in \beta N$ such that $q = pr$. For these extension relations we investigate the existence of irreducible elements, cancelation laws and properties of orders induced by these relations.

We hope that this will lead to better understanding of some problems of number theory.

Predrag Tanović

Mathematical Institute SANU and
Faculty of Mathematics, University of Belgrade, Serbia
tane@mi.sanu.ac.rs

Simple invariants in first order structures

We discuss two kinds of simple invariants in first order structures: cardinal numbers (like dimensions of vector spaces) and linear order-types.

Stevo Todorčević

Institut de Mathématique de Jussieu (UMR 7586) Case 247,
4 Place Jussieu, 75252 Paris Cedex, France and
Department of Mathematics, University of Toronto,
Toronto, Canada M5S 2E4.
stevo@math.univ-paris-diderot.fr

Union theorems for trees

Joint work with K. Tyros

We try to develop dual Ramsey theory of homogeneous finitelly branching trees. For example, we are able to extend the classical Carlson-Simpson theorem about partitions of ω into the context of partitions of arbitrary homogeneous finitelly branching trees of height ω.'

Thilo Weinert

Einstein Institute of Mathematics,
Hebrew University of Jerusalem, Israel
weinert@mail.huji.ac.il

On partitioning linearly ordered quadruples in canonical non-choice-contexts

The partition relation

$$\rho \longrightarrow (\sigma \vee \tau, \varphi)^{\psi} \tag{1}$$

means that, given a set X of order-type ρ for any colouring $\chi : [X]^{\psi} \longrightarrow 2$ of the subsets of X having order-type ψ in two colours there is a subset of X having order-type σ or τ which is homogeneous in colour 0 or a subset of X having order-type φ which is homogeneous in colour 1.

The negation of (1) is written

$$\rho \not\longrightarrow (\sigma \vee \tau, \varphi)^{\psi}.$$

For any order-type ρ, ρ^* is the one attained by reversing the ordering. For any two order-types ρ and σ, $\rho + \sigma$ is the type of the orderings given by an ordering of type ρ left to an ordering of type σ. E.g., $\omega^* + \omega$ is the order-type of the integers.

We specify the axiom system used for proving a theorem, BP stands for the statement that all sets of real numbers have the property of Baire.

Previous Results

In [6], Erdős, Milner and Rado proved, using the Axiom of Choice, the following three theorems.

Theorem 1 (ZFC). *$\rho \not\longrightarrow (\omega^* + \omega, 4)^3$ for any linear ordering ρ.*

Theorem 2 (ZFC). *$\rho \not\longrightarrow (\omega + \omega^*, 4)^3$ for any linear ordering ρ.*

Theorem 3 (ZFC). *$\rho \not\longrightarrow (\omega^* + \omega \vee \omega + \omega^*, 5)^3$ for any linear ordering ρ.*

New Results

Using a structural analysis from [2] by Blass it is possible to prove analogous statements in a choiceless context for ${}^{\alpha}2$ lexicographically ordered for some ordinal number α:

Theorem 4 (ZF)**.** *$\langle {}^{\alpha}2, <_{lex} \rangle \not\longrightarrow (\omega^* + \omega, 5)^4$ for any ordinal number α.*

Theorem 5 (ZF)**.** *$\langle {}^{\alpha}2, <_{lex} \rangle \not\longrightarrow (\omega + \omega^*, 5)^4$ for any ordinal number α.*

Theorem 6 (ZF)**.** *$\langle {}^{\alpha}2, <_{lex} \rangle \not\longrightarrow (\omega^* + \omega \vee \omega + \omega^*, 7)^4$ for any ordinal number α.*

One can, using a folklore variation of a theorem of Mycielski and Taylor, cf. [7], [5], [4] and [1], prove a positive result for the Cantor space by assuming that every set of reals has the property of Baire.

Theorem 7 (ZF + BP)**.** *$\langle {}^{\omega}2, <_{lex} \rangle \longrightarrow (1 + \omega^* + \omega + 1 \vee \omega + 1 + \omega^*, 5)^4$.*

Furthermore, for countable ordinal numbers α one can strengthen theorem 6:

Theorem 8 (ZF)**.** *$\langle {}^{\alpha}2, <_{lex} \rangle \not\longrightarrow (\omega^* + \omega \vee \omega + \omega^*, 6)^4$ for any $\alpha < \omega_1$.*

Similarly, one can prove the following two theorems:

Theorem 9 (ZF)**.** *$\langle {}^{\alpha}2, <_{lex} \rangle \not\longrightarrow (\omega^* + \omega \vee \omega + 2 + \omega^*, 5)^4$ for any $\alpha < \omega_1$.*

Theorem 10 (ZF)**.** *$\langle {}^{\alpha}2, <_{lex} \rangle \not\longrightarrow (2 + \omega^* + \omega \vee \omega + \omega^*, 5)^4$ for any $\alpha < \omega_1$.*

Outlook

We will close by speculating about the situation for $\langle {}^{\omega_1}2, <_{\mathrm{lex}} \rangle$ under the axiom of determinacy in light of the following theorems.

Theorem 11 (ZF + AD, cf. [7])**.** BP.

Theorem 12 (ZF + AD, Martin, 1973, cf. [3])**.** $\omega_1 \longrightarrow (\omega_1)^{\omega_1}_2$.

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[7] Jan Mycielski. On the axiom of determinateness. *Fund. Math.*, 53:205–224, 1963/1964, `http://matwbn.icm.edu.pl/ksiazki/fm/fm53/fm53120.pdf`.

Szymon Zeberski

Wroclaw University of Technology, Poland
szymon.zeberski@pwr.edu.pl

Luzin and Sierpiński sets

Joint work with Marcin Michalski (Wrocław University of Technology)

We shall construct some nonmeasurable and completely nonmeasurable subsets of the plane with various additional properties, e.g. being Hamel basis, intersecting each line in a strong Luzin/Sierpiński set. Also some additive properties of Luzin and Sierpiński sets and their generalization, I-Luzin sets, on the line will be investigated.