ESGI 99

Report

Improving defrosting procedure for a frozen dough

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Abstract

The present report is the result of a week–long workshop within the 99th European Study Group with Industry organized by Department of Mathematics and Informatics, Faculty of Sciences, University of Novi Sad, February 3–7, 2014.

Our task was to solve the problem presented by the company "Bakery Milan" whose products have a prominent place in the corresponding industry sector in Serbia. In accordance with it, "Milan" is constantly seeking for an improvement which is the main motive of taking part in this Study Group. Company aims to improve existing production processes using advanced mathematical tools and to find more sophisticated ways of performing defrosting procedure in order to provide the best possible quality of the products after the heat treatment.

This working group considered defrosting procedure since the quality of final products is mainly determined by it. Our main goal was to find time of defrosting at the room temperature for a group of frozen products depending on their shape, size and a presence of yeast. We modeled dough defrosting procedure using the heat equation and Stefan problem - a moving boundary problem. We carried out numerical simulations using Matlab.

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Chapter 1

Model for two different types of bakery products

In this report we examine defrosting of a dough on different temperatures – at the room temperature and at the fridge temperature. Procedure that will be described here can be applied as model for a ”žu–žu” dough (slab shaped) and for a roll dough. We divided process of heating of the dough in several steps. In the first step we observe heating of the dough boundary where the heat from the environment is transferred by convection. Further, we assume that the heat is transferred from liquid parts to frozen parts by conduction. We also assume that the dough density ρ is equal for liquid phase and frozen phase.

In the sequence we will briefly describe all physical processes and thermo –physical properties of the dough. The thermal conductivity and specific heat are important properties needed in the analysis of the heat transfer. There are three fundamental methods of heat transfer – conduction, convection and radiation. For our mathematical model we use heat equation in the form

$$\rho \partial_t (c_p T) = \nabla (k \nabla T),$$

where $\rho[kg/m^3]$ is the dough density, $c_p[J/kgK]$ is the specific heat capacity, $k[W/mK]$ is the thermal conductivity and $T[^\circ C]$ is the dough temperature. Specific heat c_p is a material property that indicates the amount of energy a body stores for each degree increase in the temperature, on a per unit mass basis. Thermal conductivity k is a material property that describes the rate at which heat flows within a body for a given temperature difference. We remark that during the heating procedure (phase transition) we have moving boundary problem which is called Stefan problem (for details see [1]).

First and second step – heat convection

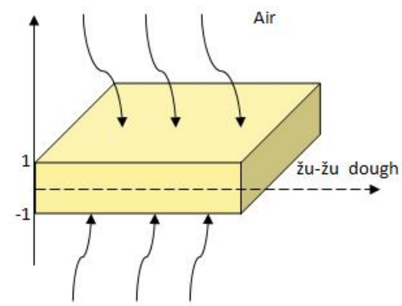

Figure 1.1: "Žu–žu" bakery product.

Convection is the transfer of energy between an object and its environment due to motion of a fluids or gases, in our case the surrounding air. In this step we have dough which is frosted at the temperature of $-10°C$ and which is placed in the room at the temperature of $20°C$ (resp. in fridge at $4°C$). We consider slab shaped type of bakery product called "žu–žu" prepared without yeast. We assume that its bottom is at the position $x = -1$ and its top is at the position $x = 1$, reducing our model to be one–dimensional (consequence of this will be that we have a change of units for ρ to [kg/m]), see Figure 1.1. Our approach is to divide the original problem into a finite sequence of (numerically) easily solvable classical problems on different domains. More precisely, instead of looking at the real mathematical problem with moving boundary (standard for phase transition problem) we will discretize the space variable and let the boundary (which is an outer ice region that is in the melting state) go along that grid.

Let the interval $[-1, 1]$ be split by an equidistant mesh of mesh step size h, where h is small enough. The heat transfer between the air and the dough takes place by free convection. At the beginning, the initial condition is $T(t = 0) = -10°C$. The boundary conditions are given by $T(x = \pm 1) = T_{surf}$ and by the following heat flux equation

$$k_{\text{amb}}(T_{\text{amb}} - T_{\text{surf}}) = \pm k_{\text{fro}} \frac{\partial T}{\partial x}\Big|_{x=\pm 1},$$

where $k_{\text{amb}}[W/m^2K]$ is the heat transfer coefficient for free convection, T_{amb} is the environment temperature and k_{fro} is the thermal conductivity of the frozen part of dough. Aim of the first step is to find the time t_1 (the surface defrosting time) such that $T(x = \pm 1, t = t_1) \geq 0$.

In the second step we have $T(t = t_1)$ as the previously obtained initial condition. Now the aim is to find time t_2 after which the dough part of height h will be defrosted. For this purpose we need to solve the heat equation on $[t_1, t_2] \times [-1 + h, 1 - h]$ for t_2 still to be determined. On the boundary of the defrosting region we have temperature zero and thus the boundary conditions $T(x = \pm 1 \mp h) = 0$. To find t_2 we use energy balance, more precisely we calculate the net flow of thermal energy into the defrosting region, which is the left hand side of the following equality

$$\int_{t_1}^{t_2} k_{\text{amb}}(T_{\text{amb}} - \underbrace{T_{\text{surf}}}_{=0\,^\circ C}) + k_{\text{fro}} \frac{\partial T}{\partial x}(-1 + h) dt = L\rho h \,,$$

the right hand side being the amount of energy needed for melting the dough layer of the height h. Here ρ is the dough density, while $L[J/kg]$ is the latent heat, i.e. the energy released or absorbed by a body or a thermodynamic system during a constant–temperature process. Note that, due to symmetry, we only wrote the condition for the lower melting region. In the interval $[1 - h, 1]$ the condition is the same apart from a change in sign in the last term on the left hand side since the spatial derivative has to be taken in the inward direction.

Further steps – heat conduction

Heat conduction is the transfer of heat energy by microscopic diffusion and collisions of particles or quasi–particles within a body due to a temperature gradient. After time t_2 we have liquid part of the dough in the two space intervals $I_\mp = [\mp 1, \mp 1 \pm h]$. We solve the heat equation on $[t_2, t_3] \times I_\mp$ with Dirichlet boundary conditions on the inside, $T(x = \mp 1 \pm h) = 0$, and the Robin condition

$$k_{\text{amb}}(T_{\text{amb}} - \underbrace{T(x = \pm 1)}_{T_{surf}}) = \pm k_{\text{liq}} \frac{\partial T}{\partial x}\Big|_{x=\pm 1} \,,$$

for the boundary that is subject to convection. As this region has just been defrosted the initial condition is $T(t = t_2) = 0$.

For the frozen part of the dough we want to solve the heat equation on $[t_2, t_3] \times [-1+2h, 1-2h]$ with boundary conditions $T(x = \mp 1 \pm 2h) = 0$ and initial condition $T(t = t_2)$ (which we obtained from previous step). Now, our goal is to find t_3 such that following equality

$$\int_{t_2}^{t_3} -k_{\text{liq}} \frac{\partial T}{\partial x}(-1+h) + k_{\text{fro}} \frac{\partial T}{\partial x}(-1+2h)dt = L\rho h$$

holds. Again we only wrote the condition for the lower defrosting region – the one for the region $[1-2h, 1-h]$ can be formulated in the same way.

In the sequel we continue with this procedure until the center of the dough reaches temperature greater than $0°C$.

Figure 1.2 gives a graphical illustration of the method we described above. We use an even number of mesh steps h, i.e. $2/h = 2n$, and repeat the procedure n times. In every step we solve the heat equation for the frozen and liquid part seperately.

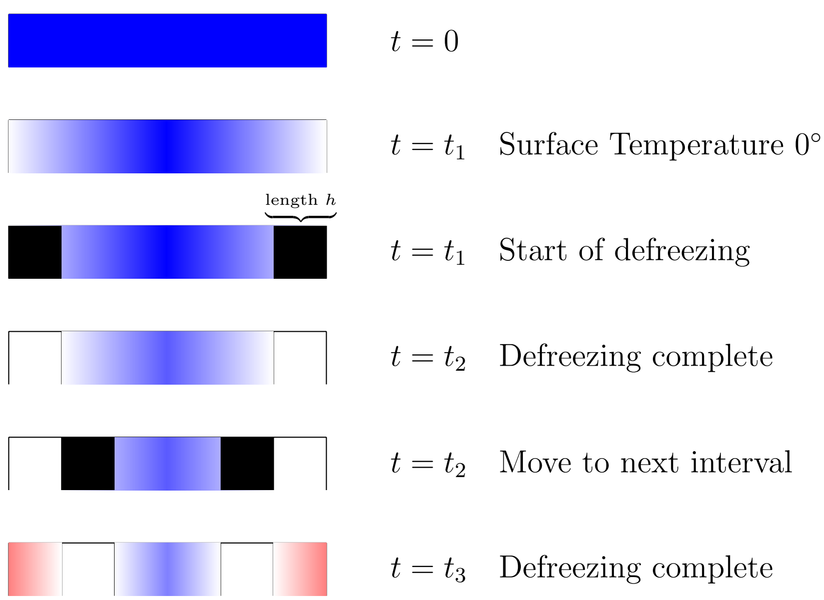

Figure 1.2: Defreezing - Black corresponds to 0° frozen, white is 0° liquid.

Numerical results for the dough without yeast

In the following we present numerical results for the defrosting of the dough. The physical parameters were taken from literature and calibrated with a series of temperature measurements on defrosting "žu–žu" that we performed. However we were not able to find a complete set of data for one specific dough and some more measurements would be necessary to calibrate the model precisely. We give results for dough thicknesses of one and three centimeters. The dough is assumed to be frosted to $-10\,^{\circ}C$ and defrost at an ambient temperature of $24\,^{\circ}C$.

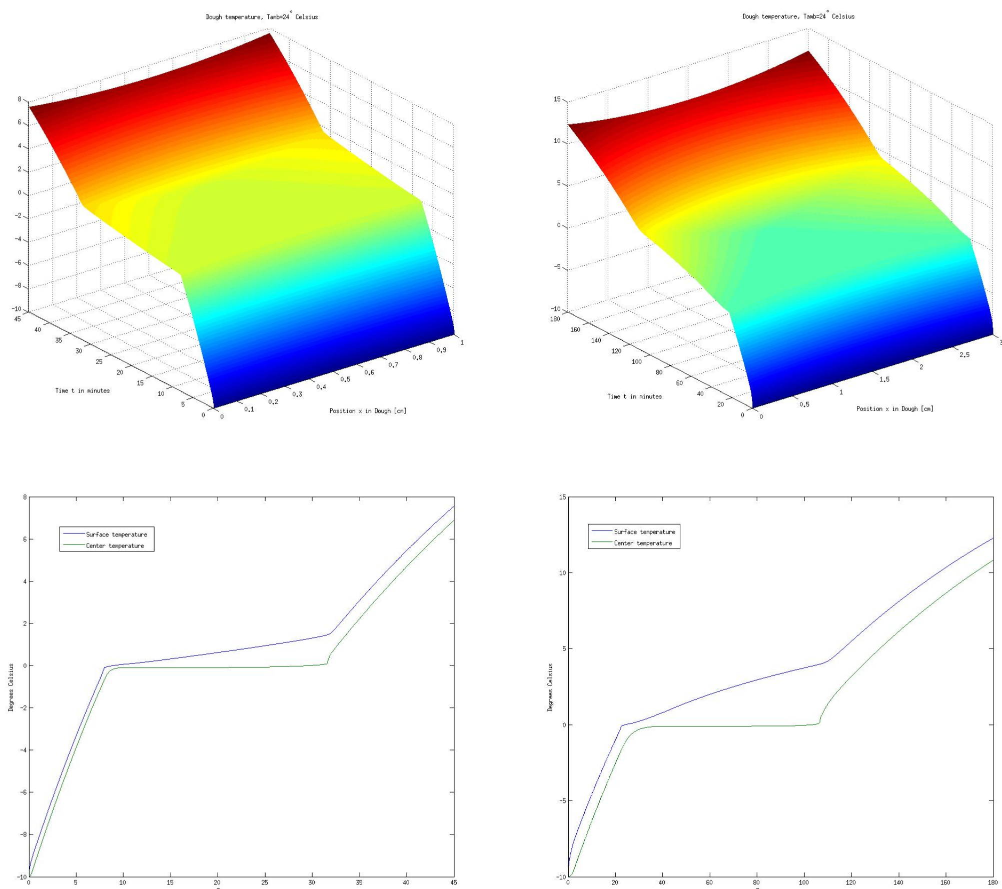

Figure 1.3: Dough temperatures during defrosting, left: $1\,cm$, right: $3\,cm$.

From our simulation we would conclude that defrosting is completed in about

35 Minutes for dough of 1 cm and in the area of two hours for dough of 3 cm thickness. One can notice that theses graph have three parts. The first part represents changing of the temperature until surface reaches $0°C$. Then, during the melting process, the curves become almost flat. In the final part the defrosted dough heats up until it reaches the ambient temperature. Here the slope is a bit smaller than in than before defrosting for two reasons. First the heat capacity of defrosted dough is larger than that of frozen dough and second the amount of energy transferred from the ambient per unit time decreases as the surface temperature approaches the ambient temperature.

This loss of energy transfer when reaching the ambient temperature becomes more apparent when defrosting at lower temperatures. In the following we give results for defrosting in the fridge at $4\,°C$. We only give the graphics

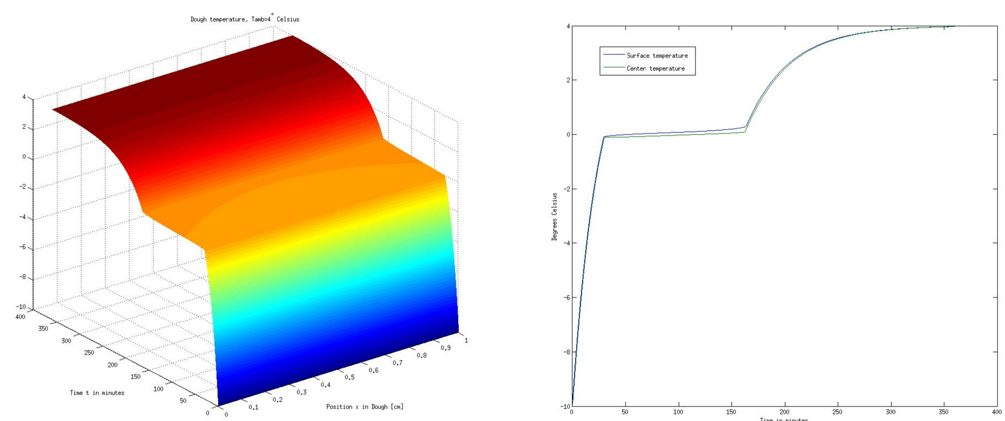

Figure 1.4: Defrosting in fridge, thickness 1 cm.

for the thinner dough since already there a time of three hours is necessary. In practical situations we suggest to leave dough in the fridge over night to defrost.

Dough with yeast

Yeast is included in different types of dough to make the dough rise and change its flavor. Since the activity of yeast fermentation depends on temperature a simple defrosting is not enough to reach the desired texture of the dough. The following graph shows the dependency of the relative activity of yeast on temperature.

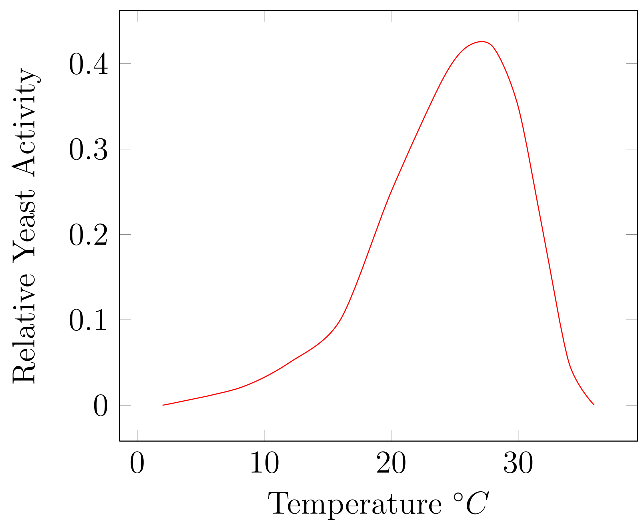

Figure 1.5: Relative yeast activity.

Yeast activity is measured by determining the amount of CO_2 that is produced. Note that enough sugar has to be provided or yeast activity will be limited by this factor.

A mathematical model for this types of dough could include the following modifications of the one we presented in the previous section

- Yeast activity depending on temperature
- Production of thermal energy due to yeast fermentation
- Amount of nutrient for yeast
- Change in thermal conductivity and density of dough during rising
- In non 1-d Situations: Increase of surface area subject to convection.

Including these into our model certainly is an interesting and, especially in the geometry-change respect challenging task. However due to time constraints we only studied the first point, *i.e.* the dependency of yeast activity on temperature. Another think to keep in mind is that inclusion of the other aspects introduces new parameters into the model that would have to be measured. Thus such a project would necessitate the collaboration with a lab.

Our simplified approach consists in determining the temperature of the dough ignoring the effects of yeast and calculating the total yeast activity at the point of lowest temperature, that is the center of the dough. This should give a lower bound on the time necessary to attain a certain level of CO_2 production in the dough depending on its thickness and serve as a starting point for experiments to determine actual values of the time necessary for defrosting and rising of the dough. Therefore we calculate the following quantity for the dough with yeast

$$Y_{total} = \int_0^{t_{\text{final}}} \mathcal{A}(T_{cent}(t))dt \geq \text{value depending of type of product}\,.$$

A dough of simmilar texture and different thickness could then be attained by taking the same value on the right hand side of the above inequality.

Numerical results for the dough with yeast

Figure 1.6 shows the temperature of a dough of thickness $1\,cm$ at room temperature of $24°C$ after two hours. About this time is necessary to reach a temperature where yeast fermentation begins to speed up.

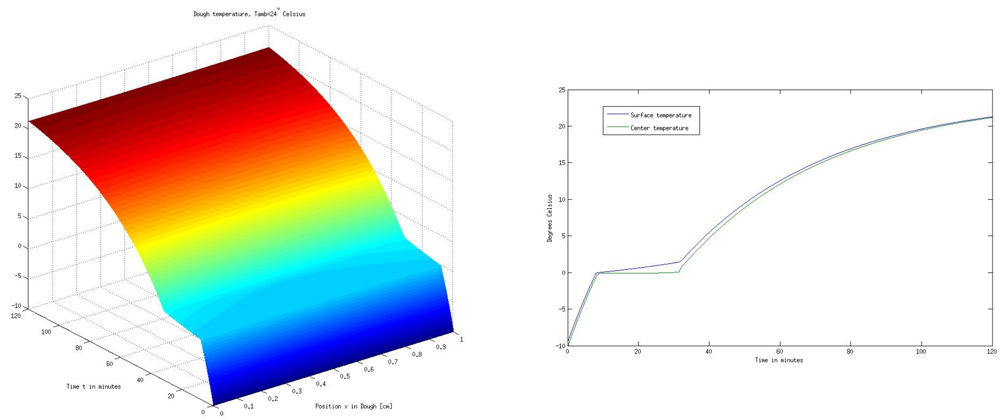

Figure 1.6: Dough temperature during two hours, thickness $1\,cm$.

From this data we calculate Y_{total} by integrating the relative activity at the current temperature over time. In Figure 1.7 we summarize these results by giving the total yeast activity that corresponds to the total amount of CO_2 produced up to three hours depending on the thickness of the dough.

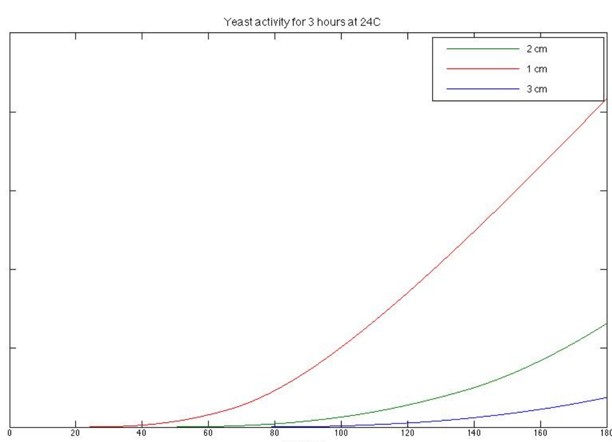

Figure 1.7: Yeast activity (dough thickness $1-3\,cm$).

We want to remark that the same model in polar coordinates could be applied to products that have a cylindrical shape, with length far greater then the diameter. This is for example the case for the bread roll of bakery Milan. The volume of the defrosting region will depend on the position but implementation is straightforward.

Chapter 2

Conclusion

In this report we present mathematical model for the dough defrosting procedure where we have concentrated on the process of changing of the dough temperature. Dough was frosted at $-10°C$ and melting process was different depending on environmental temperature. At the room temperature of $24°C$, dough reaches temperature of about 10^0C in 50 minutes. The results were obtained numerically and the rate of change of temperature is shown graphically. We obtained results for different thicknesses of dough and compared surface temperatures to the center temperature. We have the impression that for dough of thickness less than $3\,cm$ a surface temperature of $5°C$ indicates that the dough is defrosted throughout. We also obtained numerical results for dough stored in the fridge at a temperature of 4^0C. We illustrated the temperature profile over time graphically. Our results suggest that a practical recommendation for defrosting in the fridge would be leaving the dough over night.

Concerning the dough with yeast, further investigations are necessary. We just graphically presented yeast activity depending on the temperature and the dough thickness. The results were conducted in Matlab applying interpolation of a given data. Our results could be used to give a first guideline if similar products of different thicknesses are to be produced.

Acknowledgment

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References

[1] Alexiades Vasilios, Solomon D. Alan, *Mathematical modelling of melting and freezing processes*, Hemisphere Publishing Corporation, 1993.

[2] Cleland A. C., Earle R. L., *The third kind of boundary condition in numerical freezing calculations*, Int. J. Heat Mass Transfer. Vol. 20, pp. 1029-1034. Pergamon Press 1977.

[3] Cleland D. J., Cieland A. C., Earle R. L. and Byrne S. J., *Prediction of thawing times for foods of simple shape* , Int. J. Refrig., 1986, Vol. 9. July

[4] Dodić Jelena, *Optimizacija tehnološkog postupka pripreme kvasnog testa za zamrzavane pekarske proizvode*, Doktorska disertacija, Univerzitet u Novom Sadu, 2007.

[5] Lind Irene, *The Measurement and Prediction of Thermal Properties of Food during Freezing and Thawing - A Review with Particular Reference to Meat and Dough* , Journal of Food Engineering , 13 (1991), 285-319

[6] Lind Irene, *Surface Heat Transfer in Thawing by Forced Air Convection*, Journal of Food Engineering 7 (1988) 19-39

[7] Šeruga Bernarda, Budžaki Sandra, Ugarčić - Hardi Žaneta, *Individual Heat Transfer Modes During Baking of Mlinci Dough*, Agriculturae Conspectus Scientificus — Vol. 72 (2007) No. 3 (257-263)